Prior-Less Compressible Structure from Motion

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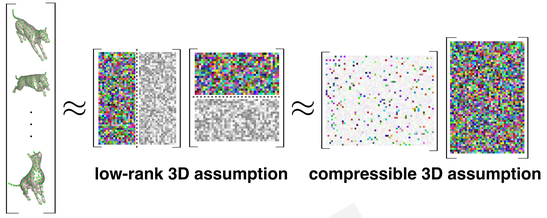
Introduction

- Re-interpret compressible SfM as a Block Sparse Dictionary Learning (BSDL) problem
- Theoretically characterize the *uniqueness* of BSDL
- Utilize the uniqueness of BSDL to recover the camera motion and 3D structures



NRSfM = BSDL?

$$\mathbf{S}^\sharp = \begin{bmatrix} x_{11} & y_{11} & z_{11} & \dots & x_{1P} & y_{1P} & z_{1P} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{F1} & y_{F1} & z_{F1} & \dots & x_{FP} & y_{FP} & z_{FP} \end{bmatrix} = \mathbf{C}^\sharp \, \mathbf{B}^\sharp$$

$$\mathbf{S} = \begin{bmatrix} x_{11} & \dots & x_{1P} \\ y_{11} & \dots & y_{1P} \\ z_{11} & \dots & z_{1P} \\ \vdots & \vdots & \vdots \\ x_{F1} & \dots & x_{FP} \\ y_{1P} & \dots & y_{FP} \\ z_{F1} & \dots & z_{FP} \end{bmatrix} = \left(\mathbf{C}^\sharp \otimes \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \right) \mathbf{B}$$

3-by-3 block sparse

dictionary

coefficients

$$\mathbf{W} = \begin{bmatrix} u_{11} & \dots & u_{1P} \\ v_{11} & \dots & v_{1P} \\ \vdots & \dots & \vdots \\ u_{F1} & \dots & u_{FP} \\ v_{F1} & \dots & v_{FP} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_1 & & \\ & \ddots & \\ & & \mathbf{M}_F \end{bmatrix} \left[\mathbf{C}^\sharp \otimes \mathbf{I}_3 \right] \mathbf{B} = \left[\mathbf{\Pi} \right] \mathbf{B}$$

2-by-3 ($\mathbf{M}_1$) … 2-by-3 ($\mathbf{M}_F$); 3-by-3 block sparse; 2-by-3 block sparse

By assuming the 3D structure $\mathbf{S}^\sharp$ is compressible, we found that the 2D measurement $\mathbf{W}$ is 2-by-3 block compressible.

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The Uniqueness of BSDL

We show that if the dictionary $\mathbf{B}$ satisfies the block spark condition*, then any feasible block sparse factorization $\{\hat{\mathbf{\Pi}}, \hat{\mathbf{B}}\}$ of $\mathbf{W}$ satisfies that

$$\hat{\mathbf{\Pi}} = \mathbf{\Pi} \left(\text{Permutation Matrix} \otimes \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \right) \text{Block Diagonal Matrix}, \quad \hat{\mathbf{B}} = \text{Block Diagonal Matrix} \left(\text{Permutation Matrix} \otimes \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \right) \mathbf{B}$$

corrective matrix

A mix inside atoms

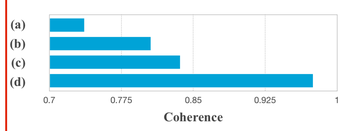
Change the ordering of atoms

*Check our paper for definitions and the the complete proof.

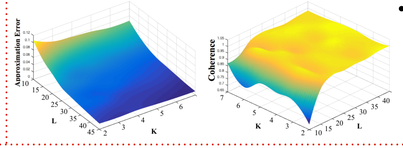
NRSfM

- Initialize BSDL by solving relaxed block ℓ_1-norm problem using Alternating Direction Method of Multipliers (ADMM).
- Solve BSDL by block K-SVD, block OMP, and block FOCUSS.
- Estimate corrective matrix.
- Estimate camera motion by enforcing camera consistency.
- Estimate 3D structure.
- Predict the confidence of the reconstruction by the coherence of the dictionary

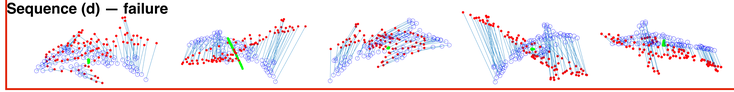
Incoherence



Control incoherence

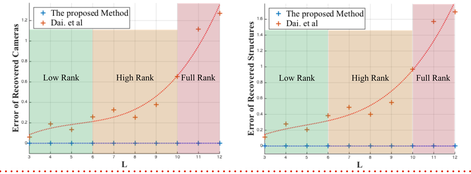


Sequence (d) — failure

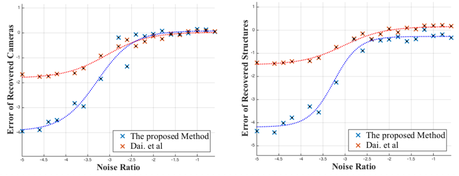


Experiments

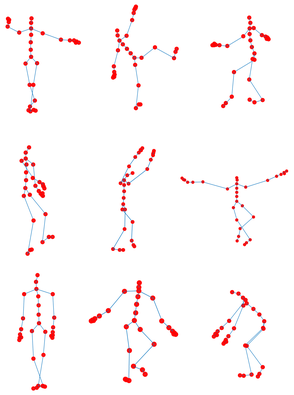
Performance vs. Rank of 3D structure



Performance vs. Noise



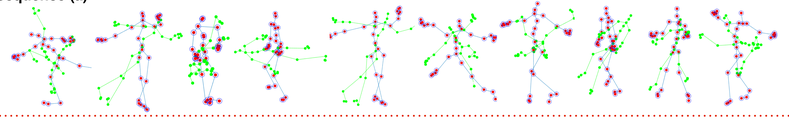
Selected recovered bases



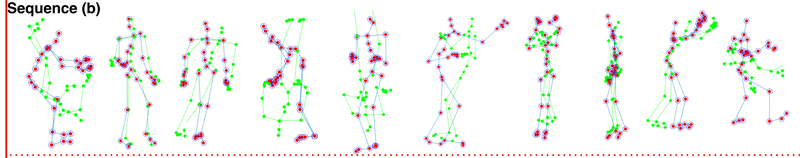
Visual performance

ground truth — The proposed — Dai. et.al

Sequence (a)



Sequence (b)



Sequence (c)